arXiv:quant-ph/0504225v1 29 Apr 2005

Comment on "Detuning effects in the one-photon mazer"

Mahmoud Abdel-Aty[1]

Mathematics Department, Faculty of Science, South Valley University, 82524 Sohag, Egypt,

In a recent work, Bastin and Martin (B-M) [Phys. Rev. A **67**, 053804 (2003)] have analyzed the quantum theory of the mazer in the off-resonant case. However, our analysis of this case refutes their claim by showing that their evaluation of the coupled equations for the off-resonant case is not satisfactory. The correct expression can be obtained by applying an appropriate formulae for the involved dressed-state parameters.

PACS: 42.50k, 32.80.-t, 03.65.Ge

The tremendous progress of ultra-cold atomic physics in the past decades is mainly due to the invention of powerful techniques for trapping and cooling atoms. The coupling between the center-of-mass motion of the atom and localized laser fields is a central topic in quantum optics, and leads to many important effects and applications in cooling, trapping, deflection, and isotope separation experiments. The interaction of ultracold atoms with microwave cavities has been considered in Refs. [1]. These studies treated the interaction between an incident atom in an excited state and a cavity field, taking the quantum mechanical center-of-mass motion of the atom into account. This interaction leads to a new kind of induced emission named the mazer action [1].

In a recent work, Bastin and Martin [2] have considered the interaction of cold atoms with microwave high-Q cavity and have argued that they have removed the restriction that considered in the previous work by considering the off-resonant case for two-level atoms. However, the two-level atoms were assumed to interact with a single mode of the cavity and the off-resonance case was considered in Ref. [3,4].

Bastin and Martin [2] have considered the Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m} + \omega_o \hat{\sigma}^+ \hat{\sigma} + \omega \hat{a}^\dagger \hat{a} + g u(z)\{\hat{\sigma}\hat{a}^\dagger + \hat{a}\hat{\sigma}^+\}. \tag{1}$$

Let us write equation (1) in the following form

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}$$

[1]E-mail: abdelatyquant@yahoo.co.uk

$$\hat{V} = \frac{\Delta}{2}\hat{\sigma}_z + \omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}\hat{\sigma}_z) + gu(z)\{\hat{\sigma}\hat{a}^\dagger + \hat{a}\hat{\sigma}^+\}. \tag{2}$$

It is easy to show that in the 2×2 atomic-photon space the eigenvalues and eigenfunctions of the interaction Hamiltonian $\hat{V}$

$$\hat{V}|\Phi_n^\pm\rangle = E_n^\pm \Phi_n^\pm\rangle, \tag{3}$$

$$E_n^\pm = (n+\frac{1}{2})\omega \pm \sqrt{\frac{\Delta^2}{4} + g^2u^2(z)(n+1)}, \tag{4}$$

$$\begin{aligned} |\Phi_n^+\rangle &= \cos\theta_n|n+1,g\rangle + \sin\theta_n|n,e\rangle, \\ |\Phi_n^-\rangle &= -\sin\theta_n|n+1,g\rangle + \cos\theta_n|n,e\rangle. \end{aligned} \tag{5}$$

In the atom-field coupling inside the cavity is considered to be a constant along the propagation axis of the atoms, the problem can be solved analytically. In this case, the mesa mode function is given by $u(z) = 1$ for $0 < z < L$, where L is the length of the cavity in the z-direction.

Now let us look more carefully at the general case, i.e. we go beyond the mesa mode case. In this case the orthonormal functions $|\Phi_n^\pm\rangle$ in the 2×2 system diagonalize the Hamiltonian and its elements are diagonal in this set of functions with

$$\begin{aligned} V_n^\pm &= E_n^\pm \\ cot2\theta_n &= -\frac{\frac{\Delta}{2}}{gu(z)\sqrt{n+1}}. \end{aligned} \tag{6}$$

The states $|\Phi_n^\pm\rangle$ are z-dependent through the trigonometric functions, they satisfy

$$\begin{aligned} \frac{\partial}{\partial z}|\Phi_n^\pm\rangle &= \pm|\Phi_n^\mp\rangle\frac{d\theta_n}{dz}, \\ \frac{\partial^2}{\partial z^2}|\Phi_n^\pm\rangle &= \pm|\Phi_n^\mp\rangle\frac{d^2\theta_n}{dz^2} - |\Phi_n^\pm\rangle\left(\frac{d\theta_n}{dz}\right)^2. \end{aligned} \tag{7}$$

Then $|\Psi(z,t)\rangle$ can be expanded in the form $|\Psi(z,t)\rangle = \sum_n C_n^\pm|\Phi_n^\pm\rangle$ and it satisfies the Schrödinger equation

$$i\frac{\partial}{\partial z}|\Psi(z,t)\rangle = \hat{H}|\Psi(z,t)\rangle. \tag{8}$$

Hence the coefficients $C_n^\pm(z,t)$ satisfy the coupled equations

$$\frac{\partial C_n^+}{\partial t} = \left(-\frac{1}{2m}\frac{\partial^2}{\partial z^2} + V_n^+ - \left(\frac{d\theta_n}{dz}\right)^2\right)C_n^+$$

$$
\begin{aligned}
& -\left(2\frac{C_n^-}{\partial z}\left(\frac{d\theta_n}{dz}\right)+C_n^-\left(\frac{d\theta_n}{dz}\right)^2\right) \\
\frac{\partial C_n^-}{\partial t} \quad = \quad & -\left(-\frac{1}{2m}\frac{\partial^2}{\partial z^2}+V_n^- -\left(\frac{d\theta_n}{dz}\right)^2\right)C_n^- \\
& +\left(2\frac{C_n^+}{\partial z}\left(\frac{d\theta_n}{dz}\right)+C_n^+\left(\frac{d\theta_n}{dz}\right)^2\right). \qquad (9)
\end{aligned}
$$

These equations should replace equations (5a) and (5b) of the B-M paper [2]. But once $u(z)$ is taken to be constant, then $\frac{d\theta_n}{dz}$ will vanish and we get the results of [3].

Also, it is important to point out that, in the last line of equation (5a) of B-M paper, $\theta\hbar\delta\sin 2$ should be replaced by $\hbar\delta\sin 2\theta$.

The most serious point is that Bastin and Martin have overlooked the formulae for $\cos 2\theta_n$ and $\sin 2\theta_n$, as well as

$$
\begin{aligned}
\cos 2\theta_n &= \frac{-\Delta/2}{\sqrt{\frac{\Delta^2}{4}+g^2u^2(z)(n+1)}}, \\
\sin 2\theta_n &= \frac{gu(z)\sqrt{n+1}}{\sqrt{\frac{\Delta^2}{4}+g^2u^2(z)(n+1)}}. \qquad (10)
\end{aligned}
$$

Once these formulae inserted in the corrected equations (5a) and (5b) of B-M paper, we find that, the second terms vanish identically and hence the results of B-M paper reduce to the same equations which have been presented in Ref. [3,4], in which we have described in details the off-resonant case on the mazer properties.

Acknowledgments: I am grateful to Prof. Abdel-Shafy F. Obada for very constructive comments and helpful discussions.

References

[1] M. O. Scully, G. M. Meyer and H. Walther, Phys. Rev. Lett. **76**, 4144 (1996); G. M. Meyer, M. O. Scully and H. Walther, Phys. Rev. A **56**, 4142 (1997); M. Löffer, G. M. Meyer M. Schröder, M. O. Scully, and H. Walther, Phys. Rev. A **56**, 4153 (1997); M. Schröder, K. Vogel, W. P. Schleich, M. O. Scully, and H. Walther, Phys. Rev. A **56**, 4164 (1997).

[2] T. Bastin and J. Martin, Phys. Rev. A **67**, 053804 (2003).

[3] M. Abdel-Aty and A.-S. F. Obada, J. Phys. B: At. Mol. Opt. Phys. **35**, 807 (2002).

[4] M. Abdel-Aty and A.-S. F. Obada, Mod. Phys. Lett. B **16**, 117 (2002).